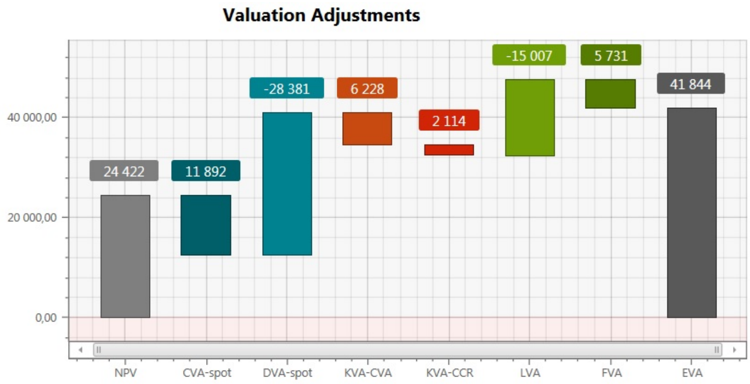
LIFETIME COSTS of xVAs

For the last decade, a significant number of valuation adjustments appeared, due to strenghtening of the regulation, as well as changes in the interbank market. These adjustments are especially relevant in the OTC market. These days, the implementation of IM-VM and of mandatory central clearing emphasizes the difficulty in assessing the actual costs of OTC contracts.
Banks already developed powerful tools to determine profiles and exposures, this means that banks can correctly evaluate the price of these adjustements at their computation date.
However, it seems that the overall profitability of such contracts depends on the value of these adjustments calculated over the lifetimes of contracts. This short note addresses key issues of this problem.

— Matthieu Maurice & Gwenaël Moysan



Capital requirements: little reminder

Basel capital requirements break up in two main components:
• $RWA(t)$, representing the risk weight calculated on the oustanding exposure at default, and multiplied by the regulatory capital ratio. The risk weight factor is based on the default probability of time t of the counterparty. The outstanding $EAD(t)$ is based on the effective rolling EPE, from which the $CVA(t)$ is deduced. This capital amount is fully relevant on the netting set. EAD calculation also depends on the choice between standard approach (CEM, SA-CCR) *v.s.* internal approach.
• $K - CVA(t)$, representing the hypothetic losses in the CVA. It can be interpreted as a 1Year VaR 99% of the CVA. This capital is calculated over all counterparties, but there is a fairly good asymptotic approximation for its value over a single counterparty, when the protfolio is large enough.
• In addition, there are refinements, like double default approach, adjustment for SME, additionnal multiplier for large financial entities, etc. All these capital requirements will be somehow remunerate, typically at the Capital Return Hurdle.

Funding Adjustments

Previously CSAs, now regulatory IM-VM and mandatory central clearing (IRS) force banks to provide large amounts of collateral, possibly high quality collateral, like cash, or applying non negligible haircuts to other types. We can distinguish:
• $FVA(t)$ funding valuation adjustments, representing some collateral costs for partially uncollateralized trades.
• $LVA(t)$ representing the funding cost of collateral, depending on the CSA, and depending also on the Initial Margin Calculation (diverse methods).
• External Central Clearing costs, that must be allocated on deals.

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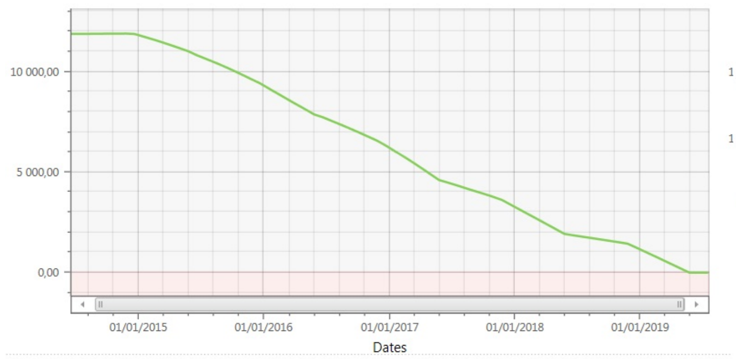
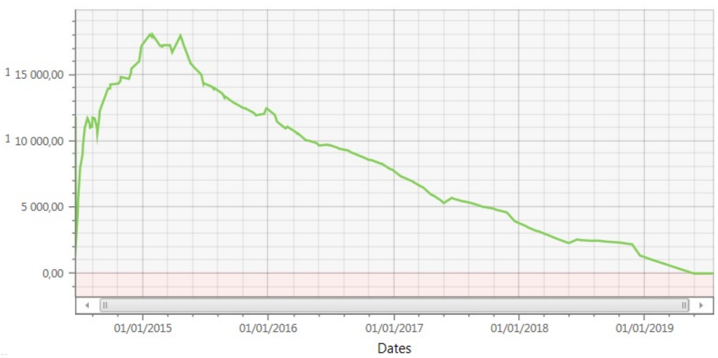


No Regression



Kernel Regression

Example of CVA profile: IRS

Lifetime Costs

To be able to evaluate the profitability of a deal, all these previous costs must be evaluated over the life of the contract, and discounted at the appropriate rate(s). At the as-of-date t_0, trades exposures $E_{t_0}(t)$ are usually computed by a Monte-Carlo framework. Adjustments are then derived from default probabilities $pd_{t_0}(t)$ computed at the same date t_0. To derive correct the lifetime cost of Basel capital requirements, we need to know the values of the exposure at a future time: what could be the value of $E_{t_1}(t)$ with $t_1 > t_0$, knowing $E_{t_0}(t)$?

An answer would be to run a Monte-Carlo at time t_1 on each of the paths of the previous Monte-Carlo. This represents a high computationnal cost, and is no longer possible if we intend to achieve this at each period of the trade.

Concrete Solutions

To assess future exposures profiles:

• A first approximation and fast computable approximation is to not account for the information available between t_0 and t_1. In this case, we have: $E_{t_1}(t) = E_{t_0}(t)$. Besides the "loss" of information, this approach does not account for the possible credit rating migration of the counterparty. The main question is, how does this approximation work?

• A better apprimation is to conduct a regression of the time t values $E_{t_0}(t)$ on the new "origin" vector: $E_{t_0}(t_1)$. Precisely, Kernel regressions allow to estimate the conditionnal expectation of a variable. Assuming some kind of Markovian behavior, we get to: $\mathbb{E}(X_t|\mathcal{I}_{t_1}) \approx \mathbb{E}(X_t|X_{t_1})$. Then we can apply the well-known Nadaraya-Watson K kernel regression:

$$\mathbb{E}(X_t|X_{t_1} = x) = \frac{\sum_i X_t^i K_h\left((X_t^i - x)\right)}{\sum_i K_h\left((X_t^i - x)\right)};$$

where h is the bandwidth, representing the appropriate range for the value of the random variable X_t. Then we can choose the Kernel function and the bandwidth associated. For example, the Gaussian Kernel is:

$$K_h(x) = \frac{\exp(-(x/h)^2/2)}{\sqrt{2\pi}}.$$

Depending on the distribution of the exposures, and the type of diffusion, we could choose a different Kernel.

• To get futures values of default probabilities, the most practical way is to run a diffusion of ratings. Together, we can get to a complete set of value.

Conclusion

Implementations of the aforementioned techniques lead to different results depending on trades exposures. On average, it seems that correclty estimating forward values of prudential ratios moves lifetime costs by a significant factor. For this topic, Global Market Solutions provides consultancy and software solutions / components integration, feel free to contact us.

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